TECHNICAL NOTES 3

Hydraulic Classifiers

3.1 Classification Based on Differential Settling - The Hydrocyclone

3.1.1 General principles of the operation of the hydrocyclone

The principle of operation of the hydrocyclone is based on the concept of the terminal settling velocity of a solid particle in a centrifugal field. The conditions in an operating hydrocyclone can be described by reference to Figures 3.1 and 3.2. The feed enters tangentially into the cylindrical section of the hydrocyclone and follows a circulating path with a net inward flow of fluid from the outside to the vortex finder on the axis. The circulating velocities are very high and these generate large centrifugal fields inside the hydrocyclone. The centrifugal field is usually high enough to create an air core on the axis that often extends from the spigot opening at the bottom of the conical section through the vortex finder to the overflow at the top. In order for this to occur the centrifugal force field must be many times larger than the gravitational field.

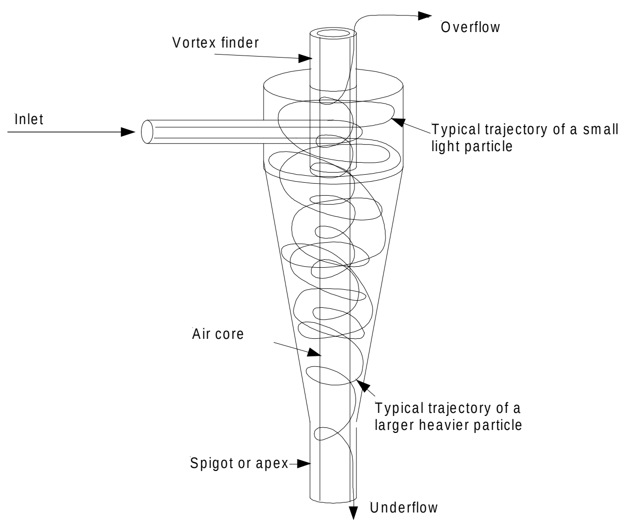

Figure 3.1 *Typical particle trajectories in a hydrocyclone.*

Particles that experience this centrifugal field will tend to move outwards relative to the carrier fluid because of their relatively greater density. The larger, heavier particles will migrate rapidly to the outside walls of the cylindrical section and will then be forced to move downward on the inside of the conical wall. Small, light particles, on the other hand will be dragged inwards by the fluid as it moves toward the vortex finder. The drag force experienced by any particle will be a complex function of the hydrodynamic conditions inside the hydrocyclone and the shape and size of the particle.

3.1.2 The Equilibrium Orbit Hypothesis

The classification action of the hydrocyclone is determined by the net effect of the two competing forces that act on every particle - the outward centrifugal force and the inward drag force. A rough guide to the effect of various operating variables on the performance of the device can be established using the so-called equilibrium orbit hypothesis. Any particle that experiences an equilibrium between these two forces inside the hydrocyclone will have an equal chance to exit through either the underflow or the overflow because they will tend to circulate on a circular orbit in the hydrocyclone and will be moved toward one or other outlet by random impacts with other particles and the random eddy motion in the highly turbulent flow field inside. An orbit on which a particle experiences a balance between the centrifugal and drag forces is called an equilibrium orbit.

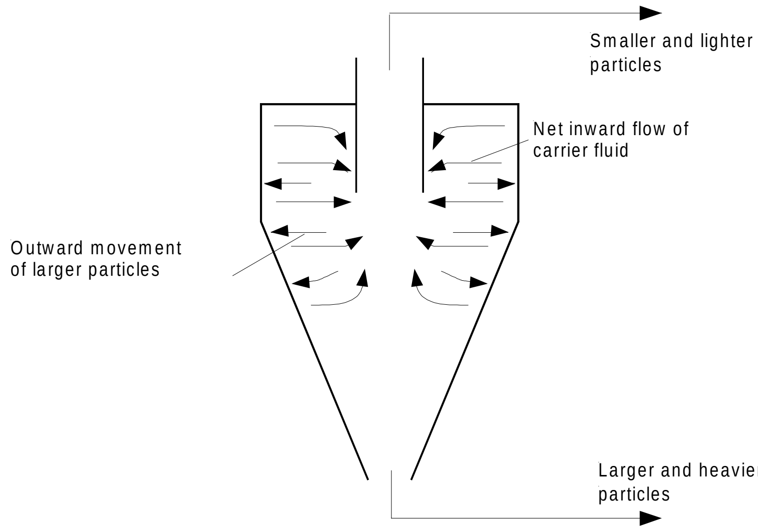

Figure 3.2 *Schematic representation of the net flow of water and the counter flow of larger particles in the hydrocyclone.*

The conditions that define an equilibrium orbit can be defined as follows: If v_r is the radial velocity of the fluid at a point in the hydrocyclone and u_r the radial velocity of the particle, the drag force is give by equation (3.9).

$$\text{Drag force} = 0.5C_D(v_r - u_r)^2 \rho_f A_c \tag{3.1}$$

where ρ_f is the fluid density, A_c the cross-sectional area of the particle and C_D is the drag coefficient. It is not difficult to show that, within the time taken for a particle to make a single orbit in the cyclone, the drag force is balanced by the centrifugal force due to the circulating motion and the particles move at their terminal settling velocities relative to the inward moving fluid. The centrifugal force is given by

$$\text{Centrifugal force} = \frac{v_\theta^2}{r} \upsilon_p(\rho_s - \rho_f) \tag{3.2}$$

where v_θ is the tangential component of the particle velocity vector, and r the radius of the tangential motion. υ_p the volume of the particle and ρ_s the density of the solid. Balancing these forces

$$0.5C_D(v_r - u_r)^2 \rho_f A_c = \frac{v_\theta^2}{r} \upsilon_p(\rho_s - \rho_f) \tag{3.3}$$

It is usually assumed that particles that have a 50% chance of passing to overflow will establish an equilibrium orbit somewhere within the cyclone. The position of this equilibrium orbit is not precisely defined although some authors claim that it is at the point where the locus of zero vertical velocity meets the spigot opening. The uncertainty regarding the actual position of the equilibrium orbit of the 50% particle is not important since we are interested only in establishing a functional form for the relationship among the particle properties that give it a 50% chance of leaving in either the overflow or underflow. All particles that have combinations of density and size that produce a 50% split in the cyclone are assumed to have equilibrium orbits at the same location in the cyclone and this assumption allows a useful correlation to be developed for the cutpoint as shown in the following analysis.

On an equilibrium orbit $u_r = 0$ and Eq. (3.3) can be written for a particle having a 50% chance of passing to overflow.

$$\frac{\upsilon_p}{A_c} = \frac{0.5\,C_D\rho_f}{(\rho_s - \rho_f)} \cdot \frac{r v_r^2}{v_\theta^2} \tag{3.4}$$

Equation (3.4) defines the so-called cutpoint for the hydrocyclone. This is the size of the particle that has a 50% chance of leaving in either the underflow or overflow. The cutpoint is normally represented by the symbol d_{50} and for spherical particles

$$\frac{\upsilon_p}{A_c} = \frac{2}{3} d_p \tag{3.5}$$

so that

$$d_{50c} = 0.75 \frac{\rho_f C_D}{\rho_s - \rho_f} \frac{r v_r^2}{v_\theta^2} \tag{3.6}$$

For a particular cyclone the particles that have a 50% chance of passing to overflow will satisfy Eq. (3.6) at a particular value of $\dfrac{r v_r^2}{v_\theta^2}$. These particles define the midpoint of the partition curve as shown in Figure 3.3 and Eq. (3.6) provides a correlation for the cut point as a function of particle properties.

The key to the equilibrium orbit theory is the drag force experienced by a particle due to the relative motion between it and the fluid. It is known that these relative velocities are small and the particle Reynolds number is usually so low that it is commonly assumed that the drag force can be calculated from the formula for the slow relative motion between a sphere and a Newtonian fluid. This is the well-known Stokes formula. This approach neglects two important phenomena: the highly turbulent nature of the fluid inside the hydrocyclone and the relatively high concentration of particles. Thus the use of Stokes flow theories which assumes that only a single isolated particle is present is not very realistic.

Four limiting cases can be considered: isolated particles in a laminar flow field (Stokes regime), isolated particles in a turbulent flow field (Newton regime), interacting particles in a laminar flow field (Blake-Kozeny regime) and interacting particles in a turbulent flow field (Burke-Plummer) regime).

The drag coefficient in Eq. (3.6) is a function of the particle size and shape and the environment that the particle finds on its equilibrium orbit. Each of the four environments defined above give different expressions for C_D.

Stokes Regime

In this regime the particles do not interact with each other and are surrounded by fluid in laminar motion. The drag coefficient is given by equation 3.25

$$\begin{aligned} C_D &= \frac{23}{Re_p} \\ &= \frac{23\mu_f}{d_p(v_r - u_r)\rho_f} \\ &= \frac{23\mu_f}{d_p v_r \rho_f} \quad \text{on an equilibrium orbit} \end{aligned} \tag{3.7}$$

Substituting in Eq. (3.6)

$$\begin{aligned} d_{50c} &= \frac{17.25\,\mu_f}{(\rho_s - \rho_f)}\frac{}{d_{50}}\frac{rv_r}{v_\theta^2} \\ d_{50c}^2 &= \frac{K_1}{(\rho_s - \rho_f)}\frac{rv_r}{v_\theta^2} \end{aligned} \tag{3.8}$$

Newtonian Regime

In this regime isolated particles are surrounded by fluid in turbulent motion and from the Abraham equation

$$C_D = 0.28 \tag{3.9}$$

Substituting in equation (3.7)

$$\begin{aligned} d_{50c} &= 0.22\frac{\rho_f}{\rho_s - \rho_f}\frac{rv_r^2}{v_\theta^2} \\ d_{50c} &= \frac{K_2}{(\rho_s - \rho_f)}\frac{rv_r^2}{v_\theta^2} \end{aligned} \tag{3.10}$$

The two limiting cases for the equilibrium orbit hypothesis (equations 3.8 and 3.10) indicate that the variation of d_{50c} with the density difference and with the volumetric feed rate should take the form

$$d_{50c} = \frac{KD_c^k}{(\rho_s - \rho_f)^m Q_c^n} \tag{3.11}$$

where *m* and *k* are constants having a values between 0.5 and 1 and n is a constant between 0 and 0.5. The lower limit of 0.5 for *m* and *k* represents laminar flow conditions and turbulence inside the hydrocyclone will give rise to higher values of *m.* and *k* and lower values of *n.*

Experimental observations have shown that the ratio v_r/v_θ is independent of total flowrate at every point inside the hydrocyclone and the assumption of turbulent flow (equation 3.10) indicates that d_{50c} should be at most a weak function of the total flowrate while the assumption of laminar flow conditions (equation 3.8) indicates that d_{50c} should be inversely proportional to the square root of the volumetric flowrate. Under turbulent conditions, d_{50c} may be expected to increase linearly with the cyclone diameter.

No one set of assumptions is likely to describe the operating behavior of the hydrocyclone under all conditions. Consequently only comparatively crude empirical correlations are currently available for the prediction of the variation of d_{50c} with hydrocyclone geometry and operating conditions.

3.1.3 Empirical Performance Models for Hydrocyclones

The technology of comminution is intimately connected with classification devices through the concept of closed-circuit milling. All comminution operations are not selective in that they can potentially reduce the size of all particles in the unit. It is undesirable to reduce the size of any particle beyond the desired product size for that unit since that consumes additional energy and further down-stream processing can be adversely effected. A classifier placed at an appropriate point in the circuit can selectively remove all of these particles that meet the product size criteria for the circuit and return coarse particles back to the comminution unit.

Unfortunately no classifier operates perfectly and classifiers will not divide a population of particles into two groups separated at a definite and particular size. All classifiers are characterised by a distribution function which gives the efficiency of separation at any size and the distribution function is more or less

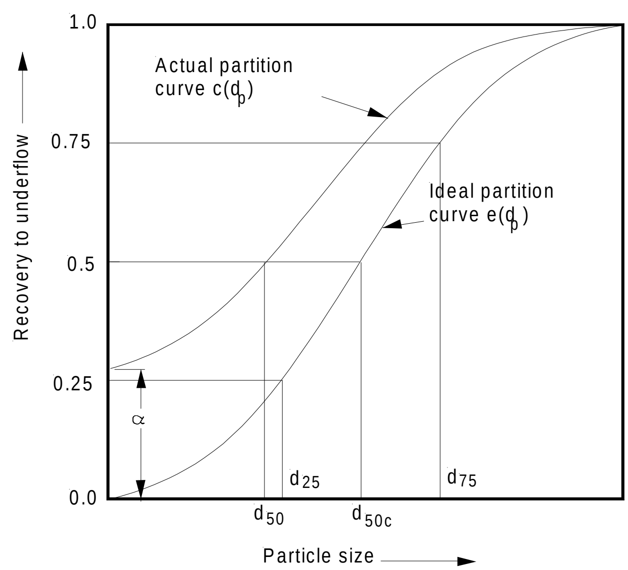

Figure 3.3 *Typical partition curves for a hydrocyclone.*

sharp depending on the efficiency of separation. A typical classification curve is shown in Figure 3.3. This is called the partition curve (sometimes called the Tromp curve) and it shows the fraction of particles at a particular size that will be partitioned to the coarser fraction. The S shaped curve is typical of all practical classifiers and a variety of quantitative expressions have been used to describe the shape of the curve.

A characteristic of virtually all practical classifiers is the phenomenon of short circuiting. All classifiers exploit some physical process to separate particles on the basis of size. This will be the differential settling velocity in a viscous fluid in spiral, rake and hydrocyclone classifiers or the physical sieving action in a screening operation. Some particles pass through the equipment without being subjected to the physical separation action. In practice it is only the short circuiting of the fine particles to the coarser product stream that is significant. This shows up as a non-zero intersection on the partition axis at zero size. In the hydrocyclone this is due to the water carrying fine particles into the boundary layer on the outer wall of the conical section and discharging them with the underflow. In other classifiers such as spiral, rake and the various screening operations, fine particles are physically carried with the large particles into the coarse product stream.

This short circuit effect can easily be accounted for by reference to Figure 3.4.

If the ideal classification action of the unit is described by classification function $e(d_p)$ and a fraction α of the feed short circuits directly to the coarser product then a simple mass balance gives the actual classification curve as

$$c(d_p) = \alpha + (1 - \alpha)e(d_p) \tag{3.12}$$

$e(d_p)$ is called the corrected classification function and $c(d_p)$ the actual classification function.

The important parameter that characterizes the operation of the classifier is the size at which $e(d_p)$ is 0.5. This size is usually called the corrected d_{50c} (corrected since the effect of the short circuit flow has been eliminated as shown in Figure 3.3). d_{50c} and d_{50} are defined by the equations

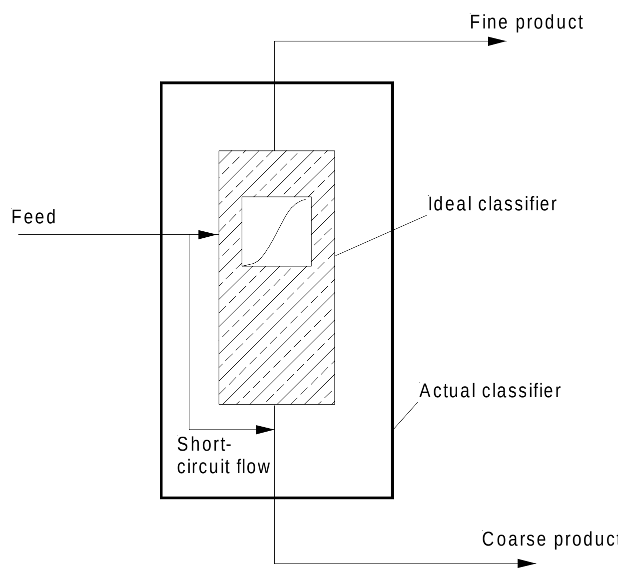

Figure 3.4 *A hydrocyclone with short-cicuit to the underflow.*

$$e(d_{50c}) = 0.5 \tag{3.13}$$

and

$$c(d_{50}) = 0.5 \tag{3.14}$$

d_{50} is clearly an indicator of the size at which the classifier cuts the particle population.

Although phenomenological models have been developed for the main classifier types, no completely satisfactory procedure exists for the calculation of the classification function in any of the main types of classifier. In the hydrocyclone the function is determined primarily by the turbulent dispersion within the centrifugal velocity field in the conical section and this has been a fertile field of research for many years. On the various types of screening devices, the classification function is determined by the kinetics of the transmission process of particles through the apertures of the screen. These processes have been comprehensively studied but it is not yet possible to make accurate calculations of the classification function in terms of the dimensions of and operating load on the classifier. However, a number of useful empirical functional forms are available to describe the ideal classification function. The most commonly used are:

1. Rosin-Rammler

$$e(d_p) = 1 - \exp(-0.693x^{\lambda}) \tag{3.15}$$

2. Exponential Sum

$$e(d_p) = \frac{\exp(\lambda x) - 1}{\exp(\lambda x) + \exp(\lambda) - 2} \tag{3.16}$$

3. Logistic

$$e(d_p) = 1/(1 + x^{-\lambda}) \tag{3.17}$$

In these equations $x = d_p/d_{50c}$ and λ is a parameter that quantifies the sharpness of the classification. It is relatively easy to appreciate the sharpness of classification in terms of the sharpness index defined by

$$SI = d_{25}/d_{75} \tag{3.18}$$

with $e(d_{25}) = 0.25$ and $e(d_{75}) = 0.75$. SI has a value between 0 and 1with low values indicating poor and inefficient separation while a value of 1.0 indicates perfect classification at the cut size d_{50c}.

The parameter λ is related to the sharpness index for each case given above as follows:

1. Rosin-Rammler

$$(SI)^{-\lambda} = 4.819$$

$$SI = \exp(-1.572/\lambda) \tag{3.20}$$

$$\lambda = \frac{-1.572}{\ln(SI)} \tag{3.21}$$

2. Exponential Sum

$$SI = \frac{\ln[(\exp\lambda + 2)/3]}{\ln[3\exp\lambda - 2]} \tag{3.22}$$

λ is usually quite large and equation (3.22) can be solved approximately for λ in terms of SI as follows

$$\lambda = 1.099\frac{(1+SI)}{(1-SI)} \tag{3.23}$$

3. Logistic

$$(SI)^{-\lambda} = 9$$

$$SI = \exp(-2.1972/\lambda) \tag{3.25}$$

$$\lambda = \frac{-2.1972}{\ln(SI)} \tag{3.26}$$

These functions are useful for the quantitative description of the behavior of the various classification units but it is necessary to evaluate the three parameters α, d_{50} and λ in terms of the physical dimensions and the actual operating conditions relevant to the unit in question.

3.1.4 The Plitt Model for the Hydrocyclone

The classification action of the hydrocyclone depends on a balance between the hydrodynamic drag forces that tend to convect the particle toward the axis and the centrifugal force that tends to move the particle outward towards the wall of the cone. The d_{50} size will be determined for the particle that finds these two forces in balance and this gives rise to the equilibrium orbit hypothesis that is discussed in section 3.1.2 . This hypothesis allows some general conclusion to be drawn for predicting d_{50} as a function of hydrocyclone geometry and operating conditions.

The performance of the hydrocyclone is strongly influenced by the short circuit to underflow and this is determined by the volumetric flow split between over- and underflow. The volumetric flow split, S, is a function primarily of the ratio of spigot to vortex finder diameters but is also a strong function of the total feedrate. As the flowrate through the hydrocyclone increases, the diameter of the air core increases choking off the underflow. This effect has not been comprehensively studied and we use a correlation developed by Plitt (CIM Bulletin, Dec. 1976, pp. 114-123) based on experimental data.

$$S = \frac{a(D_u/D_o)^b \ (D_u^2 + D_o^2)^c \ h^d \ \exp(0.54\varphi)}{D_c^f H^g} \tag{3.27}$$

where

- S = volumetric flowrate in underflow / volumetric flowrate in overflow
- D_u = spigot diameter
- D_o = vortex finder diameter
- D_c = cyclone diameter
- φ = volume fraction solids in the feed
- h = vortex finder to spigot distance
- H = slurry feed head

Plitt recommends the following values for hydrocyclone operating with free discharge a = 3.79 b = 3.31 c = 0.36 d = 0.54 f = 1.11 g = 0.24

The inverse dependence on the feed head H should be noted. As the head increases the flow through the hydrocyclone increases, the centrifugal field increases and the air core expands choking off the discharge from the spigot

The following recoveries are defined

R_v = volumetric flowrate in the underflow / volumetric flowrate in the feed

R_f = recovery of fluid phase to the underflow

R_s = recovery of solid phase to the underflow.

The volumetric recovery to underflow is related to S by

$$R_v = \frac{S}{S+1} \tag{3.28}$$

The bypass fraction α is assumed to be equal to the fraction of water that reports to the underflow R_f, and this is related to the recovery of solids R_s by

$$R_v = R_s\varphi + R_f(1-\varphi) \tag{3.29}$$

$$\alpha = R_f = \frac{R_v - R_s\varphi}{1-\varphi} \tag{3.30}$$

R_s is determined by the actual classification function which in turn is itself a function of R_f

$$R_s = \sum_i c(d_{pi})p_i^F \tag{3.31}$$

where p_i^F represents the particle size distribution in the feed.

Using equation (3.12)

$$\begin{aligned} R_s &= \sum_i [R_f + (1-R_f)e(d_{pi})]p_i^F \\ &= R_f + (1-R_f)\sum_i e(d_{pi})p_i^F \end{aligned} \tag{3.32}$$

Substitution into equation (3.30) and simplifying gives

$$R_f = \frac{R_v - \varphi\sum_i e(d_{pi})p_i^F}{1-\varphi\sum_i e(d_{pi})p_i^F} \tag{3.33}$$

Thus the hydrocyclone performance is completely determined if the corrected classification function $e(d_p)$ is known together with the volumetric flow split S.

Plitt uses the Rosin-Rammler model (3.15) to describe the partition function which requires two parameters d_{50} and λ. Plitt has correlated these parameters in terms of the cyclone geometry and the operating variables as follows

$$d_{50c} = \frac{a D_c^{\,b} D_i^{\,c} D_o^{\,d} \exp[6.3\varphi]}{D_u^{\,f} h^{\,g} Q^{\,i} (\rho_s - \rho_f)^{0.5}} \tag{3.34}$$

D_i is the inlet diameter and Q the volumetric flowrate to the cyclone.

Recommended values for the constants are $a = 2.69\times10^3$ and $b = 0.46$ $c = 0.6$ $d = 1.21$ $f = 0.71$ $g = 0.38$ $i = 0.45$. With this value of a, d_{50c} from equ tion (3.34) will be in microns. Comparison of equation (3.34) with equation (3.11) indicates that Plitt's model for d_{50c} is consistent with the main conclusions drawn from the equilibrium orbit hypothesis in that d_{50c} varies roughly in proportion to the cyclone diameter (cyclone size to the power $b+c+d-f-g = 1.18$) and inversely with feed rate to a power less than 1. The 0.5 power dependence on $\rho_s - \rho_f$ in dictates that the interaction between particles and fluid is governed by Stokes' Law but higher values can be used.

The parameter λ in equation (3.15) is correlated by

$$\lambda = a \left(\frac{D_c^2 h}{Q} \right)^{b} \exp(-1.58 R_v) \tag{3.35}$$

Recommended values for the constants are $a = 2.96$ and $b = 0.15$. This equation reveals that the efficiency of separation depends on the volumetric recovery R_v but is a comparatively weak function of the cyclone size particularly when the feed rate Q is properly matched to the cyclone size.

It is useful to convert equation (3.35) to a consistent basis for all classification functions using equation (3.21) as follows.

$$\ln(SI) = -1.24 \left(\frac{Q}{D_c^2 h} \right)^{0.15} \exp(1.58 R_v) \tag{3.36}$$

The value of the selectivity index calculated from equation (3.36) can be used in equations (3.23) and (3.26) to evaluate λ for exponential sum and logistic partition functions.

The values of the parameters a in formulas (3.27), (3.34) and (3.35) are often estimated from experimental data obtained from an existing cyclone installation in order to make the model correspond to the actual operating performance and these parameters can each be multiplied by a separate calibration factor..

In spite of the empirical nature of the Plitt hydrocyclone model it has proved to be robust for practical work. The chief source of uncertainty is in the prediction of the flow split S.

The particle size distribution in the overflow stream is given by

$$p_i^{O} = \frac{(1-c(d_{pi}))p_i^{F}}{\sum_i (1-c(d_{pi}))p_i^{F}} \tag{3.37}$$

$$p_i^{U} = \frac{c(d_{pi})p_i^{F}}{\sum_i c(d_{pi})p_i^{F}} \tag{3.38}$$

It often happens that the different minerals present in the solid have different densities. Then the classification function will be a function of the particle size and the particle composition and the size distribution density in the overflow will be given by

$$p_i^{O} = \sum_j p_{ij}^{O}(d_p,g) \tag{3.39}$$

$$= \frac{\sum_j (1-c(d_{pi},g_i))p_{ij}^{F}(d_p,g)}{\sum_i \sum_j (1-c(d_{pi},g_i)p_{ij}^{F}(d_p,g)} \tag{3.40}$$

The recovery of solids is given by

$$R_s = R_f + (1-R_f)\bar{\rho}_s^{F} \sum_i \sum_j \frac{e(d_{pi},g_j)}{\rho_s(g_j)}\; p_{ij}^{F}(d_{pi},g_j) \tag{3.41}$$

where $\bar{\rho}_s^{F}$ is the average density of the solids in the feed and $\rho_s(g)$ is the density of a particle of composition g.

Equation (3.33) becomes

$$\alpha = R_f = \frac{R_v - \varphi\bar{\rho}_s^{F}\sum_i \sum_j \frac{e(d_{pi},g_j)}{\rho_s(g_j)}\; p_{ij}^{F}(d_{pi},g_j)}{1 - \varphi\bar{\rho}_s^{F}\sum_i \sum_j \frac{e(d_{pi},g_j)}{\rho_s(g_j)} p_{ij}^{F}(d_{pi},g_j)} \tag{3.42}$$

3.2 Capacity Limitations of the Hydrocyclone

The capacity of the hydrocyclone is essentially limited by the ability of the spigot opening to discharge solids. If the solids load is too high the solids content of the underflow increases and the viscosity of the discharged pulp becomes high. The tangential velocity of the slurry at the spigot decreases and the usual umbrella-shaped discharge becomes a rope-like stream of dense pulp. Under these conditions the cyclone is said to be roping and the classifier performance is significantly degraded with the discharge of significant quantities of oversize material through the overflow. Hydrocyclones should not normally be operated in the roping condition and they are often installed in clusters to spread the feed over several units to ensure that no one cyclone is overloaded. When a cyclone is operating properly the discharge should flare out in an umbrella shape..

Preliminary selection of hydrocyclones is usually done using manufacturers charts. These show regions of operation of each cyclone in the manufacturers range as a function of the inlet pressure and the volumetric flowrate that must be processed. Typically a range of cut sizes is associated with each hydrocyclone. A typical chart is shown in the figure. It is generally not possible to use these selection charts effectively until an estimate of the circulating load is known. It is however a simple matter to make a tentative selection of an approximate cyclone size and then determine the number of cyclones in the cluster to handle the required load. Efficient simulation is the most efficient way to achieve a satisfactory design.